

Accuracy of the Transverse Velocity

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Abstract

The accuracy of the stellar transverse velocity in km/s, expected with Gaia, is estimated by combining the accuracy in annual proper motion and parallaxes. This note explains how this combination is done to generate plots as a function of the magnitude or the distance for a relevant sample of stellar types.

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References

Jordi, C.; Gebran, M.; Carrasco, et al., 2010, Gaia broad band photometry, A&A, vol. 523.

1 Introduction

The expected accuracy of the astrometry with Gaia is known for each of the five astrometric parameters (position, parallax and annual proper motions), and the latest updates is available online here `http://www.rssd.esa.int/index.php?project=GAIA&page=Science_Performance`. In these assessments all known instrumental effects have been included using the expected operating conditions (temperature, CCD operating mode, dead-time, etc.). All error sources are included as random variables with typical/average/expected deviations. In the following I refer to these pages as the *Performances Pages*. Using the performances in parallax and proper-motions, and adding known astrophysical properties of stars, one can also compute the transverse velocity $V_{\rm T}$ of each star and assess their expected accuracy, by propagating the expected astrometric performances. How this is actually done to produce reference plots is explained in the following sections of this note.

2 Notations

TABLE 1: Various notations used throughout this document

Symbol	Meaning
$\mathcal{A}$	A velocity of 1 AU per year, or $4.74047\,\mathrm{km\,s^{-1}}$
$V_{\rm T}$	Transverse velocity of a star, usually in $\mathrm{km\,s^{-1}}$
μ	Annual proper motion of a star, in mas or muas
ϖ	Stellar parallax, in mas or muas
d	Distance of a star in pc or kpc
G	Gaia broad band magnitude
l	Galactic longitude
A, B	Oort's constants
$M(X)$	Absolute magnitude in band X

3 Transverse velocity

For a star with a parallax ϖ and annual proper motion μ, the transverse velocity in $\mathrm{km\ s^{-1}}$ is given by,

$$V_{\rm T} = \mu\, d \tag{1}$$

if μ is in rads $^{-1}$ and the distance d in km. Using more convenient units one has the practical formula,

$$V_{\rm T} = \mathcal{A}\frac{\mu}{\varpi} \tag{2}$$

where $\mathcal{A}$ is a velocity of 1 AU per year, *i.e* $4.74047\,{\rm km\,s^{-1}}$, μ the annual proper motion (a pure angle), and ϖ the parallax, also an angle. If the two angles are expressed in the same unit, then V_T in (2) is in ${\rm km\,s^{-1}}$. Here I take the angles in mas or muas.

Gaia will provide the value of the annual proper motion and that of the parallax, and their associated uncertainties. With (2) it will be easy to compute the actual transverse velocity and use the radial velocity to get the three components of the velocity vector. It would be an interesting property that the uncertainties in each of the three components should be of the same order of magnitude, a feature which is not guaranteed given the two independent sources of information: astrometry for the transverse motion and spectroscopy for the radial component.

The uncertainties in μ and ϖ will be given in the form of 1-σ uncertainty. Assuming that the underlying error distributions for these two quantities are normal, one could in principle derive the probability distribution for $V_{\rm T}$, using standard results for the distribution of the ratio of two random variables, a rather intricate piece of algebra. However a much simpler approach can be used provided,

$$\frac{\sigma_\varpi}{\varpi} \lesssim 0.3 \tag{3}$$

which applies for Gaia, to 10 kpc for all stars brighter than $G = 15.5$, and to 3 kpc for stars up to $G = 18$. This is sufficient for the purpose of producing diagrams of accuracy for the transverse velocity. A more involved approach should be developed to cover all the Gaia stars to 10 kpc. The results in the final diagrams outside these ranges, must be considered as less reliable. A Monte-Carlo simulation would be the simpler way to improve the results for these extreme situations.

With the assumption (3), the variance can be propagated by linearisation, and it is known that this will lead also to the variance of the probability distribution. The condition expressed by (3) ensures essentially that the denominator of (2), is not approaching zero, even for a significant departure of the measured value from its true value.

Now, expanding to first order of the errors one has,

$$\frac{\delta_{V_{\rm T}}}{V_{\rm T}} = \frac{\delta\mu}{\mu} - \frac{\delta\varpi}{\varpi} \tag{4}$$

which lead to the variance,

$$\sigma^2_{V_\mathrm{T}} = V^2_\mathrm{T}\left(\frac{\sigma^2\mu}{\mu^2} + \frac{\sigma^2\varpi}{\varpi^2}\right) \tag{5}$$

when correlation between proper motion and parallaxes are ignored (they must be in fact negligible). Similarly by inserting (2), and taking the square root one get an estimate of the standard deviation,

$$\sigma_{V_\mathrm{T}} = \mathcal{A}\frac{\sigma_\mu}{\varpi}\left[1 + \left(\frac{\mu}{\varpi}\frac{\sigma_\varpi}{\sigma_\mu}\right)^2\right]^{1/2} \tag{6}$$

The term outside the bracket is the contribution of the uncertainty of the proper motion to that of the transverse velocity, whereas the contribution of the uncertainty in the parallax appears in the second term inside the brackets.

We know that for a mission like Gaia, running continuously over 5 years, one must have at the end between the uncertainty in the annual proper motion and parallax a rather constant ratio given by,

$$\frac{\sigma_\varpi}{\sigma_\mu} \simeq 2 \tag{7}$$

This is a pure number, since μ is always referred to as the annual proper motion. The second factor, namely μ/ϖ is of astronomical origin and given with (2) by $V_\mathrm{T}/\mathcal{A}$. There are now tow limiting regimes to consider: the nearby stars and the more distant stars.

3.1 Nearby stars

For a relatively nearby star, say within 1-3 kpc, the transverse velocity is primarily due to the individual velocity dispersion of each star in the Local Standard of Rest. The transverse velocities are on the average not too different from the radial velocities, and one has typically $V_\mathrm{T} \simeq 30\mathrm{km\ s}^{-1}$, or,

$$\frac{\mu}{\varpi} = \frac{V_\mathrm{T}}{\mathcal{A}} \simeq 6 \tag{8}$$

showing that

$$\left[1 + \left(\frac{\mu}{\varpi}\frac{\sigma_\varpi}{\sigma_\mu}\right)^2\right]^{1/2} \tag{9}$$

can be simply expressed as,

$$\frac{\mu}{\varpi}\frac{\sigma_\varpi}{\sigma_\mu} \tag{10}$$

3.2 Distant stars

At larger distance, the transverse motion follows primarily from the galactic shearing, when the effect of the differential rotation produces systematic motion larger than the more or less random velocity dispersion. The amplitude of this systematic effect, for stars of low galactic latitude, can be expressed to first order by the two main Oort's constants A and B as,

$$V_\mathrm{T} = (A\cos 2l + B)d \tag{11}$$

where d is the distance in kpc, l the galactic longitude, $A \sim 15$ km s^{-1} kpc^{-1}, and $B \sim -12$ km s^{-1} kpc^{-1}. The first term averages to zero over the galactic longitudes and we are left with the systematic transverse velocity Bd. The transition between the random proper velocity and the systematic increase due to the galactic rotation happens at $d_\mathrm{cr} \sim 2.5$ kpc. Then above this distance on has now in (9)

$$\frac{\mu}{\varpi} = \frac{V_\mathrm{T}}{\mathcal{A}} = \frac{Bd}{\mathcal{A}} \simeq 2.5d \tag{12}$$

where d is the distance in kpc, meaning that the simplification (10) still holds for distance stars.

3.3 Final expression

Finally in both regimes, the uncertainty on the parallax remains the leading factor in the propagation of the uncertainty to the transverse velocity, always at least six times larger than the contribution from the uncertainty on the annual proper motion. One can eventually write,

$$\sigma_{V_\mathrm{T}} \simeq \mathcal{A}\frac{\sigma_\varpi}{\varpi}\frac{\mu}{\varpi} \tag{13}$$

that could have been derived directly from (2) by neglecting the uncertainty in the annual proper motion. In (13), the term μ/ϖ must be taken in (8) or (12) according as the distance is below or above 2.5 kpc (in fact I have used 2.4 kpc to avoid slight discontinuity in the plot).

4 Magnitude effect

The analytical approximation of the expected accuracy in parallax given in the *Performances Pages* is as follows,

$$\sigma_\varpi(\text{muas}) = \begin{cases} 7 & \text{if } G < 12 \\ \left(9.3 + 658.1 \times 10^{0.4(G-15)} + 4.568 \times 10^{0.8(G-15)}\right)^{1/2} & \text{if } G \geq 12 \end{cases} \quad (14)$$

where I have neglected the small colour term.

Combined with

$$G(d) = M(G) + 5\lg(d) - 5 \quad (15)$$

with d in parsecs, this permits to evaluate σ_ϖ for each stellar type as a function of its distance.

For a representative sample of early and late type stars listed in Table 2, I have computed the absolute G magnitude, using the properties of the Gaia photometric system given in Jordi et al. (2010), to obtain $G - V$ as a function of $V - I$.

TABLE 2: Absolute magnitude $M(G)$ for a sample of early and late type stars. $M(V)$ and $V - I$ from Allen's Astrophysical quantities, $G - V$ computed from the Gaia photometric system properties found in Jordi et al. (2010)

	$M(V)$	$V-I$	$G-V$	$M(G)$
A0V	0.7	0.00	-0.01	0.64
A5V	2.0	0.22	-0.04	1.96
F0V	2.7	0.47	-0.09	2.61
F5V	3.5	0.64	-0.13	3.37
G0V	4.4	0.81	-0.18	4.22
K0V	5.9	1.06	-0.28	5.62
K5V	7.3	1.62	-0.54	6.76
KOIII	0.7	1.30	-0.38	0.32
K5III	-0.2	2.10	-0.83	-1.03

5 Accuracy of the transverse velocity

Combining for each stellar type of Table 2, the absolute magnitude in the G-band, the effect of distance in (15) to get G, with the standard error in parallax given by (14) and inserting in (13), one gets the expected accuracy in the transverse velocity as a function of the distance for each of the nine stellar type sampled. The results are plotted in Fig. 1 as a function of the G magnitude and in Fig. 2 for the same data as a function of the distance in pc. Labels with relevant distances or magnitudes are also introduced inside the plot area for convenience. In the diagram against magnitude, discontinuities in slope appear first at $G = 12$, resulting from the constant bright-star parallax accuracy adopted for $G < 12$, and then at different magnitudes when the distance is equal to 2.4 kpc, when the Galactic shearing takes over on the random velocity dispersion. In the second diagram, as a function of distance, this latter discontinuity is also easily spotted, while the magnitude effect at $G = 12$ shows up with the clustering of all the curves along the same line.

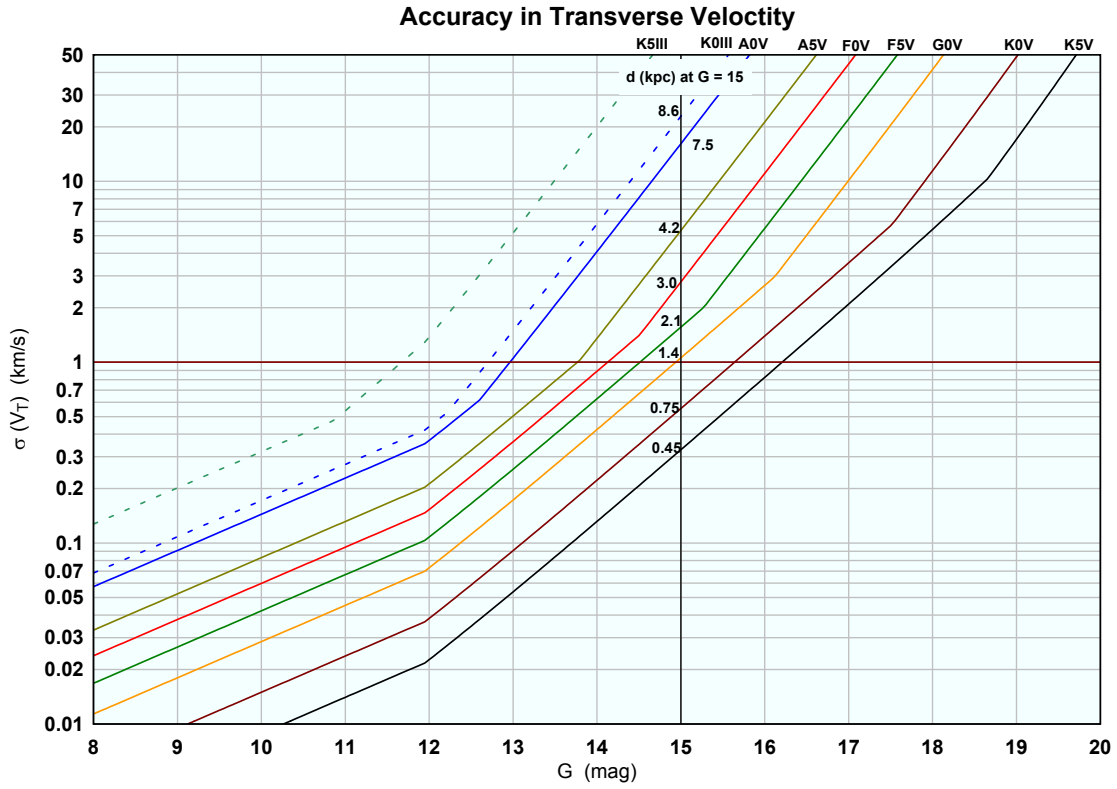


FIGURE 1: Expected performances of Gaia in the determination of the transverse velocity for a sample of representative stars, as a function of the magnitude in the Gaia broad band (the G magnitude). The uncertainties are determined from a combination of the uncertainties in the annual proper motion and parallax, as a function of the G magnitude.

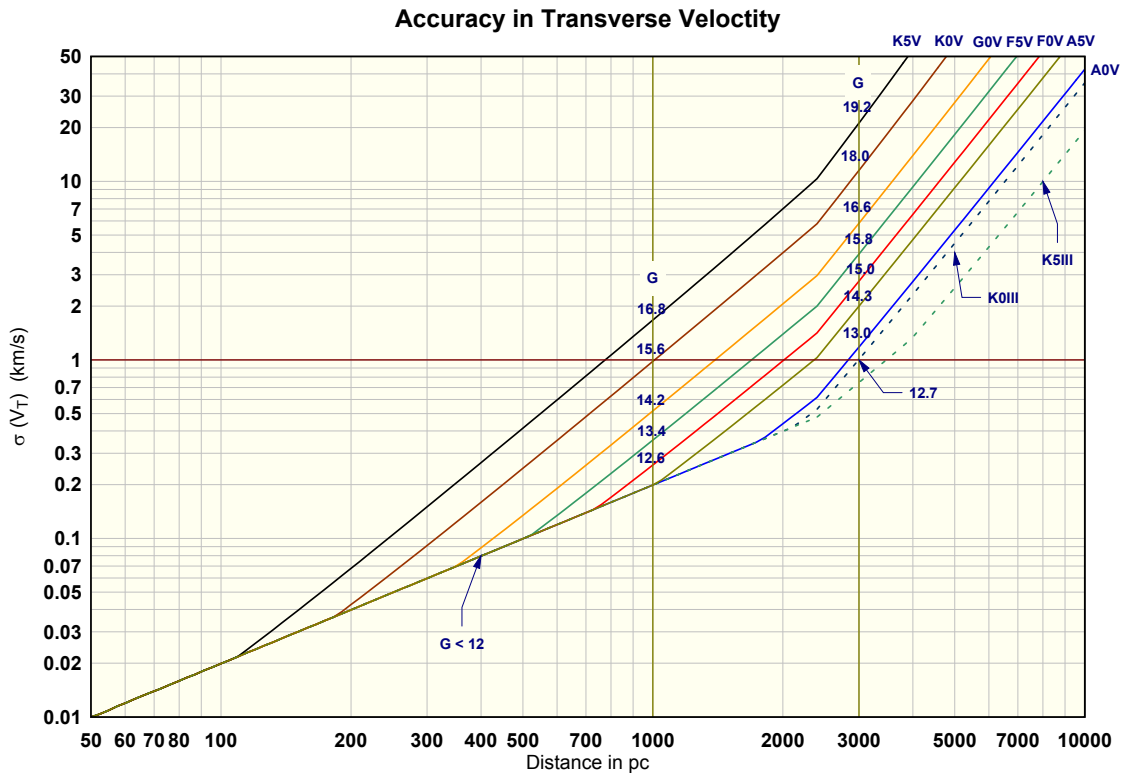


FIGURE 2: Expected performances of Gaia in the determination of the transverse velocity for a sample of representative stars, as a function of their distance. The uncertainties are determined from a combination of the uncertainties in the annual proper motion and parallax, as a function of the G magnitude.

By comparison to the expected performances in radial velocity plotted in Fig. 3 (courtesy of J. de Bruijne), one can see that globally the transverse velocity, for the core of the mission (dwarfs G to K, around $V = 15 - 16$), is better than the radial component, but not dramatically better. The RVS performance degrades quickly for fainter stars, faster than the astrometric determination of the transverse velocity and then at the faint end, we have no radial velocities at all. For the stars brighter than $V13 - 14$, the RVS performances remain rather constant, close to 1 km s^{-1}, while the transverse velocity would normally be much better, typically by one order of magnitude. Complementary ground based observations at the bright end will be very useful to improve the radial velocity and bring the accuracy at the level of the transverse velocity, a desirable feature for further kinematic studies. To survey radial velocities to 0.05-0.1 km s^{-1} for a large sample of bright stars won't be an easy task, but at least this investigation provides a quantitative input for a target performance.

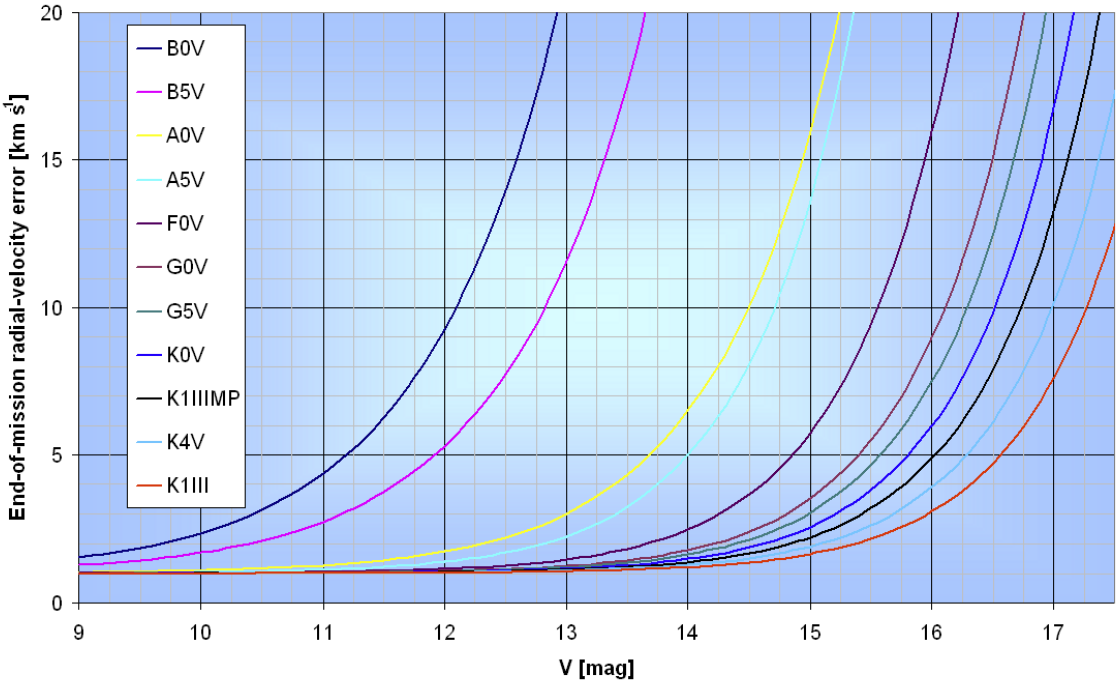


FIGURE 3: Expected performances of Gaia in the determination of the radial velocity for a sample of representative stars, as a function of their V magnitude. Plot by J. de Bruijne, available in the Gaia *Performance Pages*.